OCR
RECOGNISING ACHIEVEMENT

Friday 21 June 2013 – Morning

A2 GCE MATHEMATICS

4729/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A and B are two points on a line of greatest slope of a smooth inclined plane, with B a vertical distance of 8 m below the level of A. A particle of mass 0.75 kg is projected down the plane from A with a speed of $2\,\text{m s}^{-1}$. Find

(i) the loss in potential energy of the particle as it moves from A to B, **[2]**

(ii) the speed of the particle when it reaches B. **[4]**

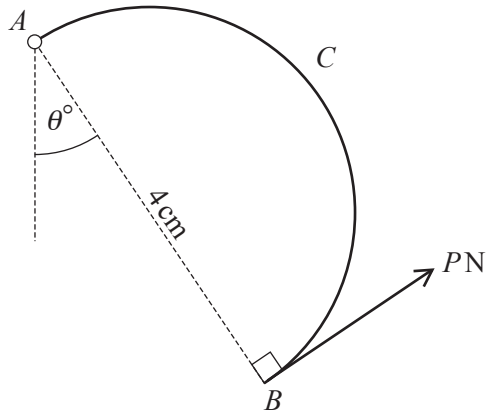
2 The power developed by the engine of a car as it travels at a constant speed of $32\,\text{m s}^{-1}$ on a horizontal road is 20 kW.

(i) Calculate the resistance to the motion of the car. **[3]**

The car, of mass 1500 kg, now travels down a straight road inclined at 2° to the horizontal. The resistance to the motion of the car is unchanged.

(ii) Find the power produced by the engine of the car when the car has speed $32\,\text{m s}^{-1}$ and is accelerating at $0.1\,\text{m s}^{-2}$. **[4]**

3



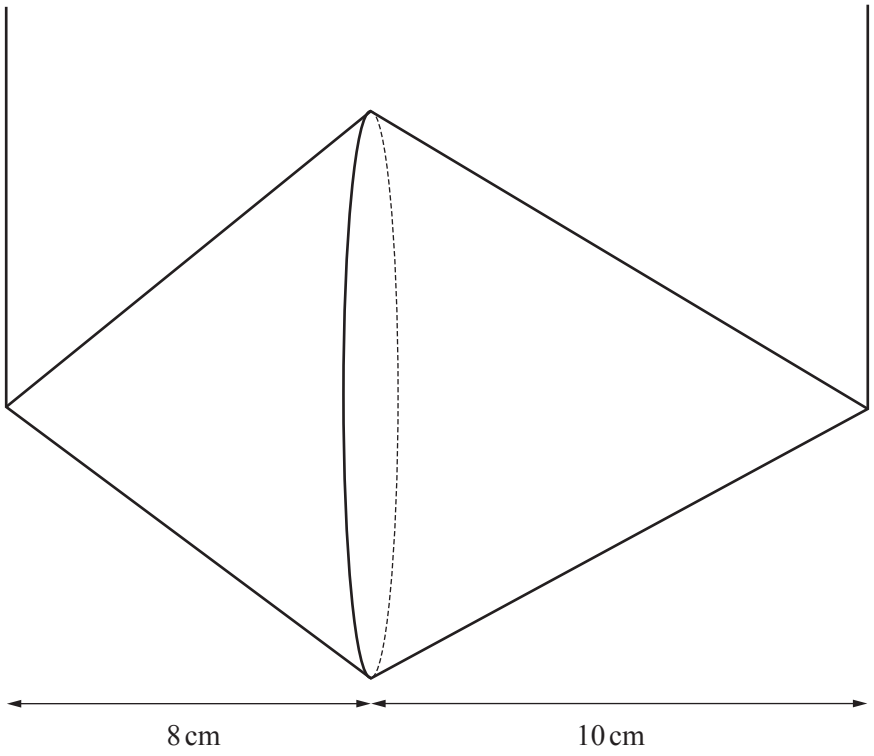
A uniform semicircular arc ACB is freely pivoted at A. The arc has mass 0.3 kg and is held in equilibrium by a force of magnitude P N applied at B. The line of action of this force lies in the same plane as the arc, and is perpendicular to AB. The diameter AB has length 4 cm and makes an angle of $\theta°$ with the downward vertical (see diagram).

(i) Given that $\theta = 0$, find the magnitude of the force acting on the arc at A. **[6]**

(ii) Given instead that $\theta = 30$, find the value of P. **[4]**

4 A solid uniform cone has height 8 cm, base radius 5 cm and mass 4 kg. A uniform conical shell has height 10 cm, base radius 5 cm and mass 0.4 kg. The two shapes are joined together so that the circumferences of their circular bases coincide.

(i) Find the distance of the centre of mass of the shape from the common circular base. **[4]**



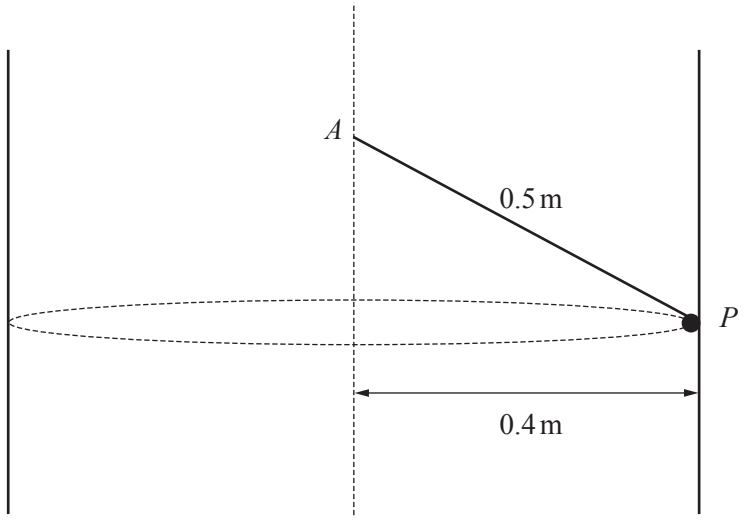
The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal (see diagram).

(ii) Find the tension in each string. **[4]**

5 A vertical hollow cylinder of radius 0.4 m is rotating about its axis. A particle P is in contact with the rough inner surface of the cylinder. The cylinder and P rotate with the same constant angular speed. The coefficient of friction between P and the cylinder is μ.

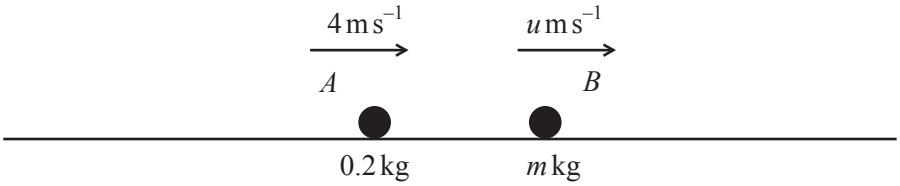
(i) Given that the angular speed of the cylinder is $7\,\text{rad}\,\text{s}^{-1}$ and P is on the point of moving downwards, find the value of μ. **[5]**

The particle is now attached to one end of a light inextensible string of length 0.5 m. The other end is fixed to a point A on the axis of the cylinder (see diagram).



(ii) Find the angular speed for which the contact force between P and the cylinder becomes zero. **[5]**

6



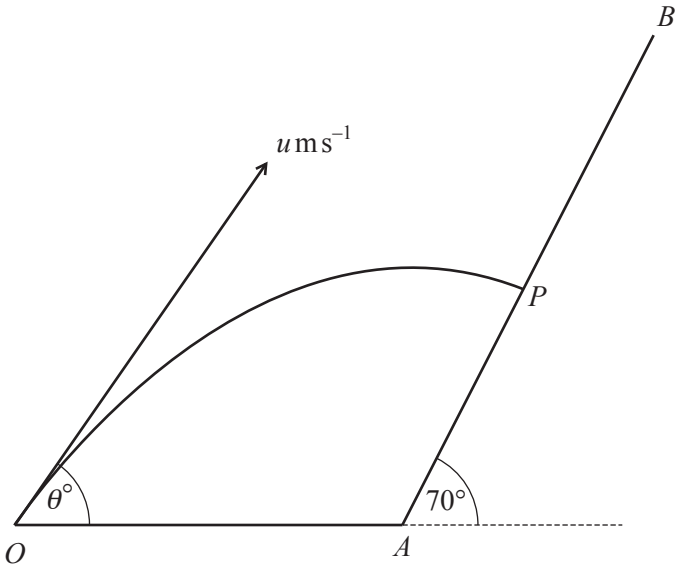
The masses of two particles A and B are 0.2 kg and m kg respectively. The particles are moving with constant speeds $4\,\text{m}\,\text{s}^{-1}$ and $u\,\text{m}\,\text{s}^{-1}$ in the same horizontal line and in the same direction (see diagram). The two particles collide and the coefficient of restitution between the particles is e. After the collision, A and B continue in the same direction with speeds $4(1 - e + e^2)\,\text{m}\,\text{s}^{-1}$ and $4\,\text{m}\,\text{s}^{-1}$ respectively.

(i) Find u and m in terms of e. **[6]**

(ii) Find the value of e for which the speed of A after the collision is least and find, in this case, the total loss in kinetic energy due to the collision. **[5]**

(iii) Find the possible values of e for which the magnitude of the impulse that B exerts on A is 0.192 N s. **[4]**

7



The diagram shows a surface consisting of a horizontal part OA and a plane AB inclined at an angle of 70° to the horizontal. A particle is projected from the point O with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of $\theta°$ above the horizontal OA. The particle hits the plane AB at the point P, with speed $14\,\mathrm{m\,s^{-1}}$ and at right angles to the plane, 1.4 s after projection.

(i) Show that the value of u is 15.9, correct to 3 significant figures, and find the value of θ. **[7]**

(ii) Find the height of P above the level of A. **[3]**

The particle rebounds with speed $v\,\mathrm{m\,s^{-1}}$. The particle next lands at A.

(iii) Find the value of v. **[5]**

(iv) Find the coefficient of restitution between the particle and the plane at P. **[1]**

Question			Answer	Marks	Guidance
1		(i)	$0.75 \times g \times 8$ 58.8 J	M1 A1 **[2]**	Weight × distance Allow –58.8
1		(ii)	 $+/-(½ \times 0.75 \times v^2 - ½ \times 0.75 \times 2^2)$ $½ \times 0.75 \times v^2 - ½ \times 0.75 \times 2^2 = 58.8$ $v = 12.7$ m s^{-1}	*M1 A1 dep*M1 A1 **[4]**	Attempt at change in KE Equate their change in KE to their PE from **(i)**
	OR	(ii)	$a = g\sin\theta$ $s = {}^{8}/_{\sin\theta}$ $v^2 = 2^2 + 2 \times g\sin\theta \times {}^{8}/_{\sin\theta}$ $v = 12.7$ m s^{-1}	B1 B1 M1 A1 **[4]**	θ is angle of slope to horizontal. Not $a = g$, not $s = 8$
2		(i)	20000/32 $R = 20000/32$ $R = 625$ N	B1 M1 A1 **[3]**	 cao
2		(ii)	 $F + 1500g\sin 2 - 625 = 1500 \times 0.1$ Power $= 32 \times F$ Power = 8380 W or 8.38 kW	M1 A1ft M1 A1 **[4]**	Using Newton 2, all forces used. ft their R from (i) SC $F - 1500g\sin 2 - 625 = 1500 \times 0.1$ Using their F. 8383.27…. SC 41200 W or 41.2 kW (41216.7…)
3		(i)	$x_G = (2 \times 2)/\pi$ P(or X) $\times 4 = 0.3g \times x_G$ $Y = 0.3g$ Use $R^2 = X^2 + Y^2$ to find R $R = 3.09$ N	B1 *M1 A1ft B1 dep*M1 A1 **[6]**	$x_G = 1.2732$…. May be seen in **(ii), mark only once.** Take moments about A or B $P = 0.9358$….. ft their x_G for this mark.

Question			Answer	Marks	Guidance
3		(ii)		M1	Attempt at moments, force × distance = $0.3g$ × distance
			$P \times 4 =$	A1	
			$0.3g \times (2\sin 30 + x_G \sin 60)$	A1	$0.3g \times 2.1026\ldots\ldots$
			$P = 1.55$	A1	1.545453…….
				[4]	
4		(i)		M1	Table of moments idea. Moments about other axes acceptable
			$4.4x_G = 4 \times ¼ \times 8$	A1	
			$\quad - 0.4 \times 1/3 \times 10$	A1	
			$x_G = 1.52$ cm	A1	Allow $^{50}/_{33}$
				[4]	
4		(ii)		M1	
			$T_{shell} \times 18 = 4.4g \times (8 - 1.52)$ or $T_{cone} \times 18 = 4.4g \times (10 + 1.52)$	A1ft	Or any other correct moment equation. ft on x_G from **(i)**
			$T_{shell} + T_{cone} = 4.4g$	M1	May use a second moments equation
			$T_{shell} = 15.5$ and $T_{cone} = 27.6$	A1	For both
				[4]	
5		(i)	Vertical force = mg	*B1	
			Horizontal force = $m \times 0.4 \times 7^2$	*M1A1	
			Uses vertical force = $\mu \times$ horizontal force	dep*M1	Dependent on B1 and M1
			$\mu = 0.5$	A1	If a value for m used B0M1A0M1A0 max.
				[5]	
5		(ii)	$mg = T \times 0.3/0.5$	B1	
				*M1	Resolve T and equate to mass × ($r\omega^2$ or v^2/r)
			$m \times 0.4\,\omega^2 = T \times 0.4/0.5$	A1	
			Solve for ω or v	dep*M1	
			$\omega = 5.72$ rad s^{-1}	A1	allow $7\sqrt{6}/3$ If a value for m and/or T used B0M1A0M1A0 max.
				[5]	

Question			Answer	Marks	Guidance
6		(i)		M1	Use of restitution, may have sign errors, must be correct ratio (v/u)
			$4 - 4(1 - e + e^2) = -e(u - 4)$	A1	
			$u = 4e$	A1	oe
				M1	Use of conservation of momentum
			$mu + 0.2 \times 4 = 0.2 \times 4(1 - e + e^2) + 4m$	A1	
			$m = 0.2e$	A1	oe
				[6]	
6		(ii)	Valid method to find e that gives the least speed	M1	Differentiate v_A and equate to 0 or complete the square on v_A
			Get $e = ½$	A1	www
				M1	Difference of KE with 4 terms
			$½ \times 0.2 \times 4^2 + ½ \times 0.1 \times 2^2 - (½ \times 0.2 \times 3^2 + ½ \times 0.1 \times 4^2)$	A1	Must have found the value of e from a legitimate method. www
			(+/–) 0.1 J	A1	SCM1A1 Loss of KE = $8e(1 - e)^3/5$ or $8e(1 - 3e + 3e^2 - e^3)/5$ or $8e/5 - 24e^2/5 + 24e^3/5 - 8e^4/5$
				[5]	
6		(iii)		*M1	Attempt to use impulse = change in momentum on one particle
			$0.2e(4 - 4e) = 0.192$ or $0.2(4 - (4 - 4e + 4e^2)) = 0.192$	A1	
			Solve three term QE in e	dep*M1	method should lead to 2 real values for e
			$e = 0.4$ or 0.6	A1	For both
				[4]	
7		(i)	$u\cos\theta = 14\cos 20$	B1	U_x =13.15... Horizontal component of initial velocity, could use U_x
				M1	Complete method to find vertical component of initial velocity, could use U_y
			$-14\sin 20 = u\sin\theta - 1.4g$	A1	$U_y = 8.9317\ldots\ldots$
			$u^2 = (1.4g - 14\sin 20)^2 + (14\cos 20)^2$	M1	Method to find u
			$u = 15.9$ **AG**	A1	cwo
			$\tan\theta = (1.4g - 14\sin 20)/14\cos 20$	M1	Method to find θ or a relevant angle
			$\theta = 34.2$	A1	SC M1A1 for $-\tan 20 = (u\sin\theta - 1.4g)/u\cos\theta$ **OR** $14^2 = (u\sin\theta - 1.4g)^2 + (u\cos\theta)^2$ B1M1A1 for both.
				[7]	

Question			Answer	Marks	Guidance
7		(ii)		M1	Method to find Level of P above A
			$\frac{1}{2} m(15.9^2 - 14^2) = mgy$	A1	
			$y = 2.9$ m	A1	
				[3]	
	OR	(ii)		M1	Use constant acc formulae, a complete method needed.
			$(14\sin 20)^2 = (15.9\sin\theta)^2 - 2gs$ or $s = 15.9\sin\theta \times 1.4 - \frac{1}{2}g \times 1.4^2$	A1ft	ft their θ from (i). no θ value used then M1A0.
			$s = 2.9$ m	A1	
				[3]	
7		(iii)	$-2.9 = v\sin 20.t - 9.8t^2/2$	B1ft	ft their 2.9
			$2.9\tan 20 = v\cos 20.t$	B1ft	ft their 2.9
			Eliminate t to obtain equation in v only	M1	Eliminate v to obtain equation in t only and solve for t
			Solve for v	M1	Substitute t to find v
			$v = 1.37$	A1	
				[5]	
	OR	(iii)		M2	Using equation of trajectory method.
			$-2.9 = (2.9\tan 20) \times \tan 20 - g(2.9\tan 20)^2/2v^2\cos^2 20$	A1ft	
			Solve for v	M1	
			$v = 1.37$	A1	
				[5]	
	OR	(iii)	$2.9/\cos 20 = \frac{1}{2}g\cos 20 \times t^2$	B1ft	
			$0 = vt - \frac{1}{2}g\sin 20 \times t^2$	B1	
			Eliminate t	M1	$t = 0.817$
			Solve for v	M1	
			$v = 1.37$	A1	
				[5]	
7		(iv)	$e = 0.098$	B1ft	ft their v from **(iii)**, must be $v/14$.
				[1]	